EVALUATION OF SMF EXPOSURE FIELD LEVELS AND GRADIENTS OBTAINABLE USING THE 2D MAGNETIC ARRAYS*

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Abstract. *Two-dimensional magnetic arrays have been proven useful as exposure setups for biomedical experiments with static magnetic fields. Different static magnetic field levels as well as vertical field gradients can be attained from these exposure setups by means of varying the geometrical parameters of an array and the type of magnetic material employed. Evaluation of obtainable field and gradient values has been conducted by varying one by one parameter. Several relevant parameters were chosen to represent the effects of input parameter changes on the magnetic flux density above the array. Calculations were conducted using the exact analytical expression.*

Key words: *Non-ionizing radiation exposures, exposure setups, static magnetic field (SMF), two-dimensional magnetic arrays, parameter adjustment*

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1. INTRODUCTION

Effects of electromagnetic (EM) fields on biological systems can be either beneficial or adversarial. Static magnetic fields (SMF) of low and moderate intensity are shown to have mainly beneficial effects, based on empirical and collected experimental data. Observed therapeutic effects include those related to treating arthritis [1], healing bone fractures [2], and improving microcirculation [3]. Mechanisms of action of SMFs are not yet fully understood.

Experimental magnetic fields are generated using various arrangements of current coils or permanent magnets. Certain types of two-dimensional (2D) magnetic arrays have been successfully employed as exposure setups for SMF generation as well [4], [5]. The type of the array described in [4], with the magnetic axes of individual magnets equally oriented and perpendicular to the array's surface, produces the slowly decreasing magnetic field. In the considered case, individual magnets were distributed across a flat surface periodically at equal distances x_d and y_d in two orthogonal directions. The dominant field component is perpendicular to the surface of the array and an order of magnitude larger than other magnetic field components, provided that individual magnets are not too sparsely placed across the surface. Magnetic flux density variation in planes parallel to the array's surface is significantly smaller than the field decrease with distance from the surface. This allows for the definition of the field gradient perpendicular to the array's surface. This exposure setup therefore produces inhomogeneous magnetic field whose magnetic flux density as well as gradient vary predominantly in the direction perpendicular to the array's surface, with very slight variations in planes parallel to the surface. This configuration enables studying the effects of both magnetic flux density and its gradient.

Different SMF field levels as well as field gradients can be attained by means of varying the geometrical parameters of an array and the type of magnetic material employed. We investigate the effects to the magnetic flux density and its gradient of varying several parameters, with the aim to define the range of SMF exposure field levels and gradients available for conducting experiments.

2. TWO-DIMENSIONAL MAGNETIC ARRAYS WITH EQUALLY ORIENTED MAGNETIC MOMENTS OF ARRAY ELEMENTS

In the investigation of obtainable exposure field levels and gradients we assume an array of N-by-N identical square cross-section magnets, equally spaced on a flat horizontal surface and kept in place by a non-magnetic substrate. We assume equal and vertical orientation of magnetic moments of all magnets. Were the magnets mounted on a ferromagnetic plate instead, similar analysis would apply, with the height of the magnets doubled due to the image theorem.

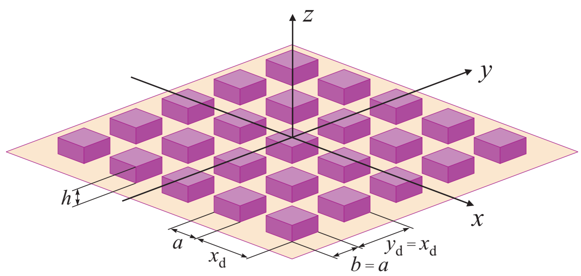

Figure 1. Two-dimensional magnetic array

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Vertical axis is denoted as the z-axis and the whole array is assumed to be symmetrical with respect to the x- and y-direction, as shown in Fig. 1. Rows of magnets are parallel to the x-axis, with the magnet centers spaced by x_d. Distance between the adjacent rows is $y_d = x_d$. Magnetic flux density distribution at the array's surface varies periodically from negative to positive B_z, since the magnetic flux lines partially close between the adjacent elements. With the increase in height, z, above the array's surface the majority of the magnetic flux lines add up together to form a resultant magnetic flux density B_z. Above a low-limit height for conducting experiments, $z = z_0$, magnetic flux density is positive everywhere except for the stray field above the array edges. With the further increase in height, magnetic flux density variation in horizontal planes decreases. We define parameter $z_{1\%}$, as the height above the array above which field variation in the horizontal planes is less than 1%, and $\overline{B}_{z,1\%}$ as the corresponding mean magnetic flux density. Magnetic field vertical gradient decreases with height as well, with magnetic field decrease almost linear at larger heights.

Basic properties of several magnetic materials most commonly used for permanent magnets [6], [7] are listed in Table 1. Material remanent magnetization, B_r, and Curie temperature, T_C, as well as the maximum energy product, $(BH)_m$, must all be accounted for when choosing the right magnets for a particular application. Improved energy product is accompanied by the increased cost of permanent magnets, ranging from about 5 USD per kg for ferrites ($BaFe_{12}O_{19}$), to about 50 USD/kg for Alnico, and to about 120 USD/kg for samarium-cobalt and neodymium magnets [8]. High quality neodymium magnets are more expensive, up to about 200 USD/kg. Remanent magnetic flux density corresponds to bulk material, i.e., to a piece of material very long in the direction of magnetization. It is related to the magnetization per unit density, M_r, by the equation $B_r = \mu_0 \cdot \rho \cdot M_r$, where ρ represents the material density. Considering equivalent surface currents resulting from magnetization and real magnet dimensions, actual magnetic flux density is obtained analytically. For the magnetization in the z-direction, four vertical sides of each magnet can be replaced by the current sheets carrying the surface current density $J_{mS} = \rho \cdot M_r$.

Vertical component of the magnetic flux density, $B_z(x,y,z)$, is calculated as the sum of the contributions of all vertical sides of all the magnets comprising the array. For a single square cross-section magnet of side length a and height h magnetic flux density is given by:

$$B_{z1}(x_1,y_1,z)=\frac{\mu_0 J_{mS}}{4\pi}\sum_{k=1}^{4}\sum_{tp=0}^{1}\sum_{tq=0}^{1}(-1)^{(tp+tq+1)}\arcsin(p_k^{tp}\cdot q_k^{tq}),$$

$$p_k^{tp}=\frac{\left((-1)^{\left[\frac{k}{2}\right]+tp}\frac{a}{2}-x_1\right)^{m_2}\cdot\left((-1)^{\left[\frac{k-1}{2}\right]+tp}\frac{a}{2}-y_1\right)^{m_1}}{\sqrt{\left((-1)^{\left[\frac{k}{2}\right]+1+(tp-1)m_2}\frac{a}{2}-x_1\right)^2+\left((-1)^{\left[\frac{k-1}{2}\right]+1+(tp-1)m_1}\frac{a}{2}-y_1\right)^2}},$$

$$q_k^{tq}=\frac{\frac{\mathrm{sgn}\left((-1)^{m_s}\frac{a}{2}-x_1\right)^{m_1}}{\left(-\mathrm{sgn}\left((-1)^{m_s}\frac{a}{2}-y_1\right)\right)^{m_2}}\cdot(z+(1-tq)\cdot h)}{\sqrt{\left((-1)^{m_s}\frac{a}{2}-x_1\right)^{2m_1}\left((-1)^{m_s}\frac{a}{2}-y_1\right)^{2m_2}+(z+(1-tq)\cdot h)^2}},$$

$$m_1=(1+(-1)^{k-1})/2,\ m_2=(1+(-1)^{k})/2,\ m_s=(1-\mathrm{sgn}(k-\tfrac{5}{2}))/2.$$

In the above, x_1 and y_1 are the distances, measured in the direction of x-axis and y-axis, from the magnet center to the field point. Magnet side is denoted by k, numbers 1, 2, 3 and 4 corresponding to the negative x-axis, negative y-axis, positive x-axis and positive y-axis with respect to the magnet center. Square brackets stand for the integer division. Point of current entrance into the current sheet corresponds to $tp = 0$, and the point of current exit to $tp = 1$. Indices $tq = 0$ and $tq = 1$ denote the bottom surface or the top surface of the magnet. Derivation of the above equation, as well as the expressions for x-component and y-component of the magnetic flux density, are given in [4]. It has been shown in [4] that the B_z field component is dominant. Therefore, this evaluation considers only the dominant field component.

Table 1. Basic properties of typical commercial magnetic materials

Material	B_r (T)	$(BH)_m$ (kJ/m³)	T_C (°C)	M_r (Am²/kg)	J_{ms} (A/m)
$BaFe_{12}O_{19}$	0.40	34	450	65.0	318.3
Alnico	1.25	43	860	142.1	994.7
$SmCo_5$	0.88	150	720	85.4	700.3
Sm_2Co_{17}	1.08	220	820	102.3	859.4
$Nd_2Fe_{14}B$	1.28	300	400	135.8	1018.6

3. Results and discussion

The surface current density, J_{mS}, enters the magnetic flux density equation as the multiplicative factor to scale the expression depending exclusively on the geometrical parameters of an array. Therefore, for a fixed given geometry of an array, difference in the achieved field levels for different magnetic materials corresponds to the ratio of remanent magnetization of materials. This is illustrated by the example shown in Fig. 2, where moderately sized magnets ($a = 8$ mm, $h = 5$ mm) were arranged with the gap between every two magnets equal to the magnet length a ($k_d = 1$). Please note that the parameter k_d is introduced as the ratio of the gap size to the magnet size. Corresponding center-to-center magnet spacing equals $x_d = (k_d+1)\cdot a$. Number of individual magnets in a row is taken equal to $N = 15$ in this as well as in the all other examples.

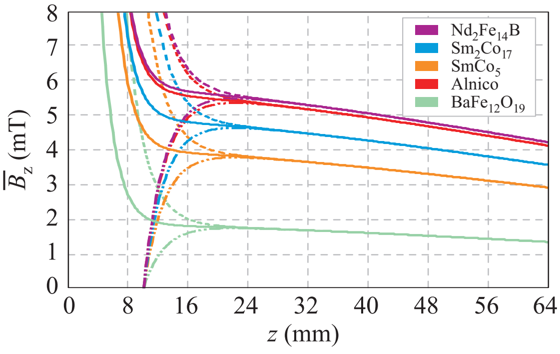

Figure 2. Magnetic flux density along the magnet axis, along the magnet gap axis, and mean magnetic flux density in

horizontal planes above the array, for magnetic materials listed in Table 1 (a = 8 mm, h = 5 mm, x_d = 16 mm)

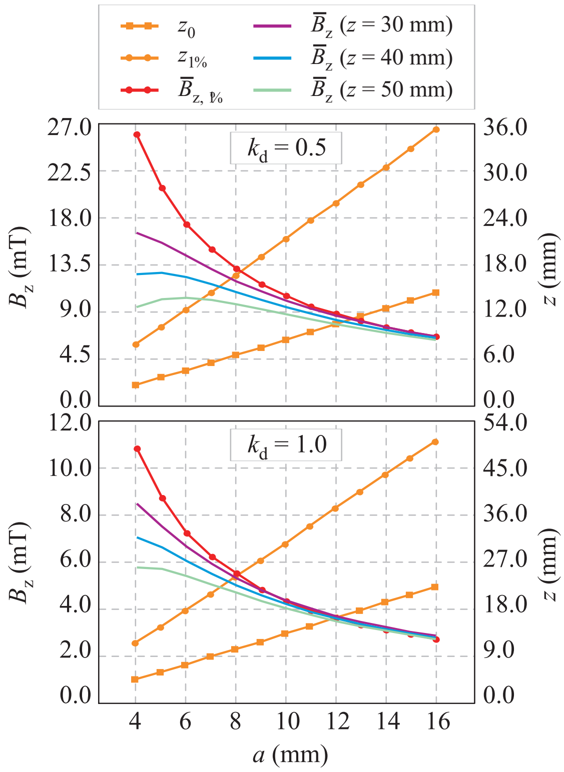

Figure 3. Influence of the magnet side length, a, on several parameters describing the magnetic field above the array, with the relative spacing between the magnets kept fixed at half the side length (upper plot) and whole side length (lower plot)

In Fig. 2, magnetic flux density at the vertical axes through the magnet centers is represented by the dashed lines, and the one between every four magnets (axes where the x-spacings and y-spacings cross) is shown by the dash-dot-dot lines. Mean magnetic flux density in horizontal planes above the array is well approximated as the average of the two (depicted by the solid lines). Up to some low-limit height magnetic flux density along the magnet gap axis is negative as the flux lines close between the magnets. At the same height, right above the magnets the field is strong, so that all in all next to the surface the field intensity is strong, direction of magnetic flux lines alternates, and field gradients are very pronounced. After the limiting height, z_0, magnetic flux densities above the magnets and between the magnets start converging to fast reach the height where the field variation everywhere in the horizontal planes lies below 1% of the mean field level in that plane. For the considered example, magnetic flux density is positive everywhere above the plane z_0 = 10.2 mm. The 1% threshold is $z_{1\%}$ = 24.0 mm, and the B_z value for the strongest neodymium magnets at that height equals $\overline{B}_{z,1\%}$ = 5.5 mT. In this particular example, field further decreases almost linearly.

Having in mind average mice height of about 30 mm, experimental volume for *in vivo* experiments can be taken from the height of about 25 mm to 55 mm. For *in vitro* experiments, a range of different field intensities is available by appropriate placement of specimens at different heights above the array. Mean magnetic flux density and its mean gradient in the experimental volume are determined by field averaging between the two limiting horizontal planes.

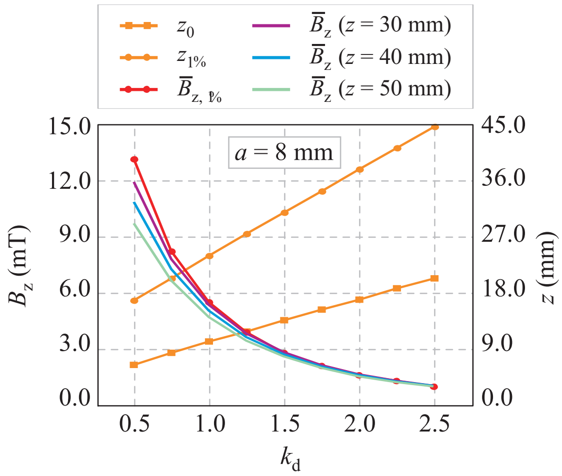

Figure 4. Influence of the gap size, $k_d \cdot a$, between the two neighboring magnets on the parameters describing the magnetic field above the array

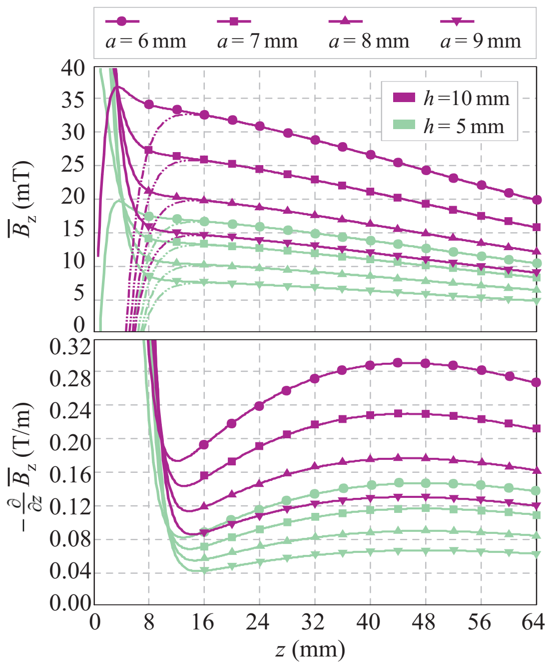

Figure 5. Mean magnetic flux density and its mean gradient for the fixed center-to-center magnet spacing of x_d = 12 mm, for different ratios a versus $k_d \cdot a$, for two magnet heights h

Since the amount of data that could be depicted in Fig. 3 and Fig. 4 is limited, three heights, z = 30 mm, z = 40 mm, and z = 50 mm, were chosen to represent field variations resulting from the changes of input parameters.

Provided that the height of the magnets is relatively small with respect to the size of an array in lateral directions, doubling the magnet height results in twice the magnetic flux density. Height of the magnets can therefore be used to adjust the field levels. If the more homogeneous magnetic flux density is desired for the experiment, one magnetic array is placed below the experimental volume, and the other one on top. Resultant magnetic flux density is fairly homogeneous. Height of the magnets in Fig. 3 and Fig. 4 is fixed at h = 5 mm and in Fig. 5 it is compared with h = 10 mm.

Data presented in Fig. 3 analyze what are the effects to the field of the changes in magnet side length, a. The ratio gap size versus magnet size is kept fixed at k_d = 0.5 (upper plot) and k_d = 1 (lower plot). Both the low-limit height z_0 and the 1% threshold $z_{1\%}$ show linear dependence on the lateral size of the magnets. Magnetic field is higher and the two limiting heights, z_0 and $z_{1\%}$, are lower for the smaller magnets. Data shown in Fig. 3, Fig. 4 and Fig. 5 correspond to $Nd_2Fe_{14}B$ magnets. For other types of magnetic materials, data need to be scaled by the relative ratio of remanent magnetizations.

Parameters of interest are presented in Fig. 4 as a function of k_d, with the fixed values of other input data. Size of the magnets is kept at moderate a = 8 mm. Similar conclusions are drawn as in the previous example – smaller magnet spacing results in the stronger and higher quality magnetic field (in terms of field homogeneity in horizontal planes).

Figure 5 shows mean magnetic flux density and its mean gradient for the fixed center-to-center magnet spacing of x_d = 12 mm, value of x_d resulting in almost constant gradient about 40 to 48 mm height. Magnetic flux density decrease in that case shows the least variation from the linear one inside the experimental volume recommended for *in vivo* experiments (25 to 55 mm height above the array). It is demonstrated that the increase in height of the magnets results in almost the same relative increase in field intensity.

4. Conclusions

Generic example of the symmetrical two-dimensional magnetic array has been studied using the analytical expressions describing the magnetic flux density above the array. Evaluation of obtainable static magnetic field levels as well as vertical field gradients has been conducted by varying one by one parameter. Input parameters comprised magnet size and spacing (the geometrical parameters) and the type of magnetic material used. The collection of data and results presented can be used for preliminary design of 2D magnetic arrays.

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